

# AdS/CFT and the cosmological constant problem

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Within the context of the AdS/CFT correspondence we attempt to formulate the cosmological constant problem in the dual conformal field theory. The fine-tuning of the bulk cosmological constant is related to an apparent fine-tuning in the effective description of the CFT in terms of its light operators: while the correlators of single particle operators satisfy a large  $N$  expansion, the expansion does not appear to be natural. Individual terms contributing to correlators have parametrically larger value than the one dictated by large  $N$  counting. The final  $1/N$  suppression of correlators is achieved via delicate cancellations between such terms. We speculate on the existence of underlying principles which might make the bulk theory (secretly) natural.

June , 2011

## 1. Introduction

Does the holographic correspondence teach us anything new about the cosmological constant problem? One of the most striking aspects of holography is that spacetime and the quantum fields that we use to describe it are emergent. This suggests the possibility that the symmetry, or mechanism, responsible for the apparent fine-tuning of the vacuum energy (and perhaps other fine-tuned couplings) may be manifest in the underlying fundamental theory, but invisible, or difficult to understand, in terms of the emergent light fields which we directly observe.

The AdS/CFT correspondence [1],[2],[3] offers a framework where this possibility can be investigated in a controlled way. In the real world we observe a positive cosmological constant and (one of) the puzzle(s) is why the observed value is so small compared to the cutoff scale of effective field theory [4],[5]. However the puzzle is insensitive to the *sign* of the cosmological constant. It would be equally puzzling if the observed c.c. was of the same magnitude but opposite sign. Hence it might be useful to try to understand the analogue of the c.c. problem in anti-de Sitter space and within the context of AdS/CFT. The latter provides us with a non-perturbative definition of AdS quantum gravity in terms of a non-gravitational quantum field theory, in which no mysteries about the vacuum energy and its renormalization should exist.

Our goal is to take a first step towards these questions by considering the analogue of the cosmological constant problem in anti-de Sitter space and translating it into some statement in the dual CFT. Ultimately we would like to ask whether the CFT can resolve the problem in a natural way. While we have not been able to fully reach these goals, we believe that this line of investigation may eventually offer some new insights.

More specifically in this paper we will try to explore the following three questions:

1. What are the properties of a CFT whose holographic dual exhibits a “cosmological constant problem”?
2. In such a theory how does the c.c. problem manifest itself in the CFT? i.e. does the CFT appear to be fine-tuned in some sense?
3. Does the CFT offer a resolution of the problem?

Let us give a brief summary of our conclusions: CFTs with holographic duals are characterized by some sort of large  $N$  expansion, which means that correlators of single-particle operators are suppressed by powers of  $1/N$ . Our main observation is that in CFTs whose bulk dual exhibits a cosmological constant problem<sup>1</sup>, the large  $N$  expansion does not appear to be natural, at least not if we try to understand it in terms of the light operators<sup>2</sup> of the CFT (in terms of glueballs and mesons, in gauge theory language). In this description it is reasonable to expand the correlators into sums of terms, each of which corresponds to the exchange of certain gauge invariant operators in intermediate channels i.e. into a sum of conformal blocks. While the sum of all these contributions is suppressed by the expected power of  $1/N$ , individual terms in the sum can be parametrically larger, as long as they cancel among themselves. We believe that these surprising cancellations are the way that the bulk c.c. fine-tuning becomes visible in the dual CFT.

We also explore the idea that the fine-tuning may be resolved and that the  $1/N$  expansion may recover a natural form, once the correlators are expressed directly in terms of the underlying fundamental fields i.e. the “quarks and gluons”. Analyzing the correlators in the language of the gauge singlets described in the previous paragraph is useful for certain purposes, for example in order to understand how the bulk dual emerges, but may obscure other properties of the theory. In particular, the  $1/N$  suppression of correlators may look natural in one description (in terms of fundamental fields) but fine-tuned in the other (in terms of gauge singlets). If this is true it means that the fine-tuning, both in the bulk and the boundary, is an artifact of trying to understand the correlation functions in terms of effective, rather than fundamental, variables.

### *Some caveats*

As we will explain later, we do not know at the moment a specific large  $N$  CFT whose dual exhibits a sharp cosmological constant problem. Whether one exists or not is an interesting question, equivalent to whether semi-classical AdS gravity without low energy supersymmetry can be UV-completed. Nevertheless we will assume that such a CFT exists and we will try to analyze what would be the CFT manifestation of the bulk c.c. fine-tuning. Lacking a concrete example to work with, our discussion will have to be

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<sup>1</sup> Assuming that such CFTs exist.

<sup>2</sup> In this paper by “light” operators we mean those of low conformal dimension.

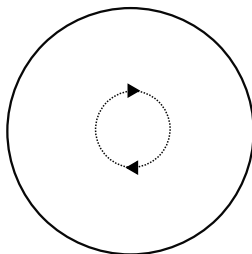
somewhat abstract and we will not be able to check whether the resolution of the problem that we sketched above does indeed take place. We hope to revisit this in future work.

Before we close the introduction we should mention that the c.c. problem in the real world is of course much more complicated than the idealized version that we consider here. Firstly, we only address one aspect of the c.c. problem which is why the observed value of the c.c. is not large, that is, not of the order of the cutoff of the effective field theory and not related questions, such as why the c.c. has the small positive value that we actually observe, or the cosmic coincidence problem. Secondly, we have ignored a class of complications: we study the case of empty, eternal anti-de Sitter space, while our universe has positive cosmological constant<sup>3</sup>, has undergone nontrivial cosmological evolution and is filled with matter. Thirdly, we only consider the fine-tuning between quantum fluctuations and bare values, while in the real world the fine-tuning is not only between these two but also additional contributions to the vacuum energy from the Higgs potential, the QCD condensate etc. Despite all these differences, we hope that some of the lessons that we will eventually learn about the c.c. problem in an idealized anti-de Sitter space may be of relevance to the problem in the real world.

## 2. The Main Ideas

### 2.1. The c.c. problem in the bulk

As we mentioned above, the main goal of this paper is to translate the cosmological constant problem into a statement in the dual CFT. A standard way to express the c.c. problem is to evaluate the 1-loop vacuum energy of matter fields. In AdS space we have to evaluate the diagram depicted in figure 1.



**Figure 1:** 1-loop contribution to the vacuum energy in AdS from a massive fermion.

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<sup>3</sup> And hence the conceptual problems of holography for de Sitter space have to be addressed.

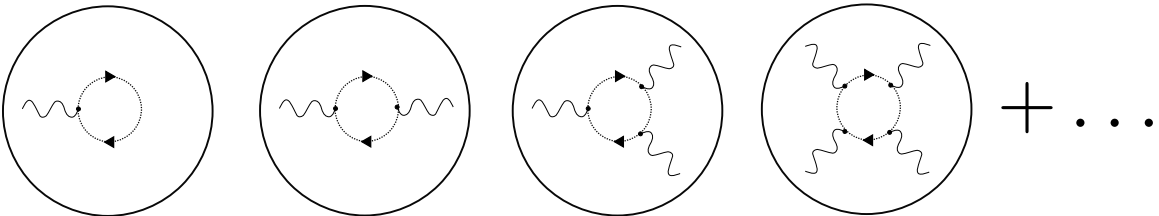
We assume that we are in a regime where classical gravity is reliable, i.e.  $R \gg 1/M_P$ , where  $M_P$  is the Planck mass and  $R$  is the AdS radius. If we call  $M_{cut}$  the cutoff of the effective field theory in the bulk, then this diagram predicts an energy density of the order  $M_{cut}^4$ , in the case of four dimensional AdS space. This energy density enters Einstein's equations multiplied by a factor of  $G_N \sim M_P^{-2}$ . Assuming that the cutoff is close to the Planck scale  $M_P$  we find that the expected gravitational backreaction of the 1-loop quantum fluctuations would lead to a modification of the AdS radius to something of the order  $R \sim 1/M_P$ .

If however, and according to our working assumption, what we actually observe is that  $R \gg 1/M_P$  it means that somehow the backreaction of the 1-loop vacuum energy got cancelled. This is usually attributed to a "counterterm" or equivalently a "bare cosmological constant"  $\Lambda_{bare}$ , and the c.c. problem is that this cancellation between the bare and 1-loop vacuum energies has to be extremely fine-tuned.

What is the interpretation of the diagram in figure 1 in the dual CFT? It is not very easy to answer this question, since in AdS/CFT we are more familiar with calculations of Witten diagrams, i.e. Feynman diagrams in the bulk where the external points are taken to infinity in AdS. Luckily, general covariance ensures that the counterterm responsible for the cancellation of the diagram in figure 1 must come in the form

$$\int d^4x \sqrt{-g} \Lambda_{bare} \quad (2.1)$$

Expanding the square root of the metric determinant in (2.1) in fluctuations around AdS we find that it generates vertices with arbitrary number of external gravitons. Indeed, these counterterm vertices are necessary to (partly) cancel the divergences of the 1-loop diagrams depicted in figure 2.



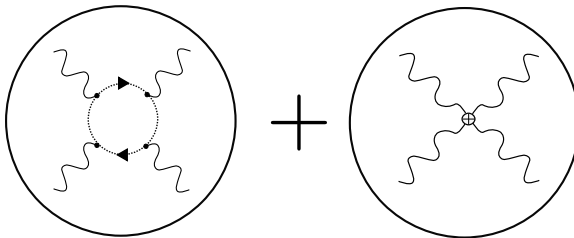
**Figure 2:** 1-loop contribution to  $n$ -point functions of gravitons.

Thus the c.c. fine-tuning at the 1-loop level is not only visible in vacuum-to-vacuum amplitudes but also in  $n$ -point functions of gravitons. Let us formulate this more precisely. The smallness of the bulk cosmological constant is equivalent to the statement that  $R \gg 1/M_P$ . This in turn means that the connected  $n$ -point function of gravitons must be suppressed, relative to the disconnected one, by a factor<sup>4</sup> of

$$(M_P R)^{2-n} \quad (2.2)$$

in the limit  $R \gg 1/M_P$ .

To be more specific consider, for example, the 4-point function of gravitons. The first nontrivial contribution to the connected 4-point function comes from the tree-level Born amplitude, which is indeed of the order  $(M_P R)^{-2}$  relative to the disconnected one, as required by (2.2).



**Figure 3:** 1-loop contribution to 4-point functions of gravitons and c.c.-type counterterm.

The situation is different however when we look at the 1-loop correction to the 4-point function due to an internal fermion loop, as shown in figure 3. Let us estimate the contribution to this diagram from the UV regime of the integration over the internal lines. We call  $p$  the momentum running in the loop (for large  $p$ ). Each of the internal propagators contributes a factor of  $1/p$  and each vertex a factor of  $p$  from the derivative coupling, which means that the diagram contains a factor of the form<sup>5</sup>

$$\int d^4 p \sim M_{cut}^4$$

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<sup>4</sup> Here we assume that the energy of the gravitons is kept fixed in units of the radius of the AdS space, as we take the  $M_P R \rightarrow \infty$  limit. The specific scaling presented here holds in four bulk dimensions.

<sup>5</sup> More details about the UV divergences of loop diagrams in AdS can be found in later sections.

The diagram is multiplied by a factor of  $G_N^2 \sim M_P^{-4}$  from the vertices. So the contribution of this diagram, relative to the disconnected one, has an overall factor of

$$M_P^{-4} M_{cut}^4$$

Taking  $M_{cut} \sim M_P$  we find that this diagram is  $\mathcal{O}(1)$  in the  $M_P R \gg 1$  limit, and thus violates the requirement (2.2) for the scaling of graviton correlators in order for the bulk theory to have a small c.c. This means that this diagram must be cancelled by a counter-term to a large extent, as shown in figure 3. The cancellation must be such that the  $\mathcal{O}(1)$  parts completely cancel giving a sum which is only of the order

$$(M_P R)^{-2}$$

To get a feeling of this fine-tuning, if we take the radius of the AdS space to be of the order of the size of the universe then we have  $M_P R \sim 10^{60}$ . Hence the two diagrams depicted in figure 3 are both of order 1 but have to be cancelled with each other, leaving behind something of order  $10^{-120}$ . This is the analogue of the c.c. fine-tuning expressed in terms of 4-point functions.

Now, 4-point functions of gravitons can be more easily translated into correlation functions in the CFT, simply by taking the external points to the conformal boundary of AdS. Then they become Witten diagrams and via the AdS/CFT prescription can be related to boundary correlators of the stress energy tensor. Following this strategy, the bulk fine-tuning can be translated into a statement of fine-tuning purely within the dual conformal field theory<sup>6</sup>.

In the rest of this paper we will try to develop this point of view.

## 2.2. Fine-tuning in the CFT

Let us now give a short summary of how we think the bulk fine-tuning manifests itself in the dual CFT. In the previous section we argued that the fine-tuning can be expressed in terms of graviton correlation functions. In AdS/CFT correlation functions of gravitons are dual to correlation functions of the stress energy tensor  $T$ . The role of the parameter

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<sup>6</sup> The reason that we did not consider the similar story for 0-, 1-, 2- or 3-point functions of gravitons has to do with the fact that it is more difficult to give the CFT interpretation of such Witten diagrams, as discussed later.

$M_P R$  is played by the central charge  $c$ <sup>7</sup>. If we redefine the stress tensor  $\tilde{T} = T/\sqrt{c}$  in such a way that its disconnected correlators are order 1, then the statement that the observed value of the bulk cosmological constant is small (as in (2.2)) is dual to the statement that the connected correlators scale like

$$\langle \tilde{T}(x_1) \dots \tilde{T}(x_n) \rangle_{con} \sim c^{(2-n)/2} \quad (2.3)$$

in the  $c \gg 1$  limit. In particular we expect that the connected 4-point function will scale like  $1/c$ . Notice that for large  $N$  gauge theories in the 't Hooft limit we have  $c \sim N^2$  and the scaling mentioned above becomes the usual  $1/N$  suppression of connected correlators.

Here is where we start to see the fine-tuning: the bulk discussion of the previous section suggests that while the 4-point function in the CFT is of order  $1/c$ , individual terms which contribute to it may be parametrically larger and it is only after summing over all of these contributions that we get the small number of order  $1/c$ . These contributions are the CFT dual of the 1-loop and “counterterm” Witten diagram depicted in figure 3. The question then is, in what sense can a general CFT correlator be thought of as being the sum of various terms. Of course we can always artificially split up a correlator into a sum of terms to make it look fine-tuned. The nontrivial question is whether there is a canonical way to do so, where these terms have some physical meaning.

Indeed, there is a canonical procedure in the CFT which corresponds to “splitting up” the correlator into a sum of factors. This is the expansion in conformal blocks

$$\langle \tilde{T}(x_1) \tilde{T}(x_2) \tilde{T}(x_3) \tilde{T}(x_4) \rangle_{con} = \sum_{\mathcal{A}} |C_{TT}^{\mathcal{A}}|^2 \mathbf{G}_{\mathcal{A}}(x_1, x_2, x_3, x_4) \quad (2.4)$$

From a CFT point of view this is a well-motivated expansion to consider and moreover is a useful intermediate step before representing the underlying quantum field theory in a holographic way: the Witten diagrams in the hologram can be related to certain (combinations of) conformal blocks with nice properties under crossing symmetry<sup>8</sup>.

All of this suggests that the bulk c.c. fine-tuning can be translated into the statement that while the LHS of equation (2.4) is of order  $1/c$ , there are terms on the RHS of the same equation which are parametrically larger and which, in the end, cancel among themselves.

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<sup>7</sup> Defined by the 2-point function of the stress energy tensor.

<sup>8</sup> See [6] for a review.



If we considered a specific AdS/CFT duality where the bulk displayed a c.c. fine-tuning problem, and if we were CFT observers, we would measure a small 4-point function, of order  $1/c$ , but if we tried to decompose it into conformal blocks we would be surprised to find that the smallness  $1/c$  is achieved via enormous cancellations of large terms from individual conformal blocks<sup>9</sup>.

### 2.3. A possible resolution of the fine-tuning

Before presenting more details, let us discuss a speculative possibility of how holography might provide us with an explanation of the c.c. fine-tuning, at least in the simplified AdS context that we consider.

In AdS/CFT, and presumably more generally in holographic theories, the degrees of freedom (fields and particles) visible to a low-energy bulk observer are not related in a simple way to the fundamental degrees of freedom. In the case of large  $N$  gauge theories the underlying degrees of freedom are quarks and gluons, while the bulk spacetime describes the dynamics of gauge singlets i.e. glueballs, mesons, hadrons etc. At the level of effective field theory in the bulk, an observer only sees the (light) gauge singlets and not the constituent gluons.

An interesting possibility is the following: the mechanism which guarantees the smallness of the c.c. (expressed by (2.3)) may be manifest when formulated in terms of the fundamental variables of the quantum field theory, for example the quarks and gluons, but invisible in terms of the low-energy fields i.e. the gauge singlets. In such a case, if we were bulk observers trying to explain the fine-tuning, we would never be able to understand its origin in terms of the bulk fields. Only if we gained access to the underlying fundamental theory (that is, the boundary QFT) would we be able to resolve it.

To rephrase, let us imagine that we start with a quantum field theory (QFT) described by a set of fundamental fields and a path integral over them. Let us moreover assume that the theory is, or flows to, a conformal field theory. Using the QFT we calculate the

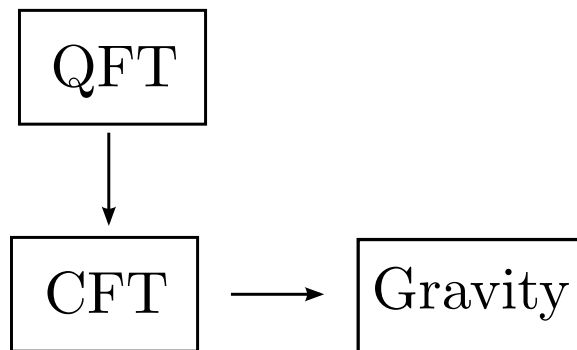
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<sup>9</sup> The reader might worry that there may exist some fundamental CFT reason that such a fine-tuning would not be possible, given bounds from crossing symmetry and unitarity, for example derived by methods such as those in [7]. However, as we mention later, we do know (supersymmetric) examples where such fine cancellations between conformal blocks take place. We thank S. Rychkov for discussions on these issues.

spectrum of conformal primaries  $\Delta_i$  and their 3-point functions  $C_{ij}^k$ . We can then describe the QFT in a more abstract way, simply as a catalogue of the CFT data

$$\{\Delta_i, C_{ij}^k\} \quad (2.5)$$

Let us call this the CFT language (CFT). If the CFT has the right properties, i.e. large central charge and a sector of few low-lying operators whose correlators (approximately) factorize, then the low-lying sector of the CFT has a natural representation as a weakly coupled theory in anti-de Sitter space. In order to describe the bulk theory we only need the CFT data of the low-lying sector expressed in the form (2.5). These data translate into the masses and couplings of what will be the light fields in AdS.



**Figure 4:** In AdS/CFT we start with a quantum field theory with many fundamental fields. Typically there are few operators of low conformal dimensions which can be described effectively as a perturbation around a generalized-free-CFT. This effective CFT is naturally represented as a higher dimensional gravitational theory. The cosmological constant fine-tuning is visible in the two lower boxes, i.e. the effective CFT and the gravitational description. It may have a natural explanation on the deeper microscopic level of the underlying QFT.

As we will try to explain in this paper, in a theory where the bulk suffers from fine-tuning, it is precisely the CFT data (2.5) that seem to be “un-natural”. If we were just given the CFT data, without the underlying QFT, we would observe large cancellations between seemingly unrelated terms, for example in double OPE expansions of the form (2.4), and we would have a hard time explaining them. These cancellations may have a natural origin in the underlying QFT due to some dynamical mechanism or symmetry. This mechanism may become invisible once we pass to the level of the abstract CFT data (2.5).

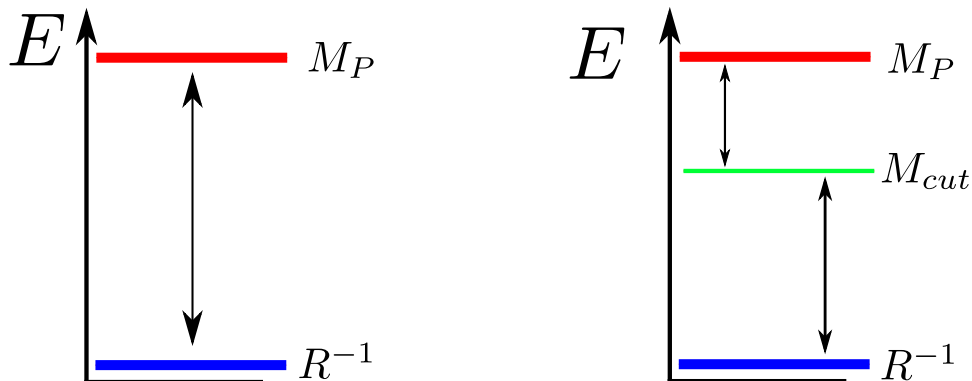
This possibility, if true, would suggest that the c.c. fine-tuning between quantum fluctuations and bare values can be translated (and resolved) purely within QFT. It can be translated into a question about the properties of the transformation that takes us from the upper box of figure 4 to the lower left box. The nontrivial property of this transformation, assuming that our speculative assumption is true, is that the  $1/c$  expansion may look natural in the first description but fine-tuned in the second<sup>10</sup>.

## 2.4. Comments

Let us make some clarifying comments:

### 1. *Is there really a cosmological constant problem in AdS/CFT?*

One might think that the existence of a dual CFT somehow removes the problem altogether, if the CFT is “stable”, for example if it has no relevant operators. But this is not the end of the story: while the existence of a dual CFT may perhaps make the problem look less dangerous, we would still like to know how the apparent bulk fine-tuning is resolved. This situation is analogous to the black hole information problem: while we do believe that the boundary CFT gives a fully unitary evolution of the system, we still try to understand how this is achieved in the language of the bulk and what is wrong with Hawking’s argument for information loss<sup>11</sup>.



**Figure 5:** (Left): the hierarchy between the Planck scale and the size of the AdS space  $M_P R \gg 1$ . (Right): perturbation theory seems fine-tuned if  $\xi \equiv M_{cut}^4 M_P^{-2} R^2 \gg 1$ .

<sup>10</sup> The two formulations are of course mathematically equivalent. Still, it is a logical possibility that certain properties i.e. the smallness of connected correlators may be manifest in one formulation but may look surprising in the second.

<sup>11</sup> We would like to thank R. Emparan for pointing out this analogy.

## 2. Hierarchy vs fine-tuning

Relatedly, one might think that the CFT interpretation of the bulk cosmological constant problem is simply that the CFT has large central charge and that this is what creates the large hierarchy. The central charge controls the parameter  $M_P R$ , so indeed the large hierarchy between the Planck and the Hubble scale is mapped to a large value of the central charge.

The point is that there is an additional puzzle, which is the main focus of this work, and it has to do with the “radiative stability” of this hierarchy from the point of view of perturbation theory. Besides the scales  $M_P$  and  $R$  another relevant scale is the cutoff scale  $M_{cut}$  up to which effective field theory in the bulk is reliable. Unless there is some mechanism (like supersymmetry) responsible for cancellations, the 1-loop diagrams up to the cutoff scale contribute a factor of  $M_{cut}^D$  to the energy density of the vacuum<sup>12</sup>. Unless there are nontrivial cancellations with the bare cosmological constant, Einstein equations predict that the size  $R$  of this space should satisfy  $M_{cut}^D M_P^{2-D} R^2 \sim 1$ . If, on the other hand we observe that the combination

$$\xi \equiv M_{cut}^D M_P^{2-D} R^2 \gg 1$$

then it means that some unexpected cancellations between loop diagrams and counterterms must be taking place. This fine-tuning is precisely the one we are interested in.

To give an example which shows that there are indeed two separate aspects of the cosmological constant problem let us compare the following hypothetical setups: imagine that we have two examples of AdS/CFT duality, both with very large central charge, both with a semi-classical gravity dual, up to a very high cutoff scale  $M_{cut}$ . Let us moreover assume that example A is supersymmetric<sup>13</sup>, while example B is non-supersymmetric. In both cases the combination  $M_P R$  is very large (since the central charge is large) and bulk observers could legitimately ask why they happen to live in a world with such large central charge.

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<sup>12</sup> Here by  $D$  we refer to the number of macroscopically large dimensions of the bulk theory. If the CFT is  $d$ -dimensional it may be that  $D = d + 1$ , or  $D$  could be larger if the duality has a large internal space, as in the case of IIB on  $\text{AdS}_5 \times \text{S}^5$ . Also, by  $M_P$  we refer to the  $D$ -dimensional Planck mass.

<sup>13</sup> For example it could be the AdS/CFT duality between the ABJM theory at large  $N$  and M theory on  $\text{AdS}_4 \times \text{S}^7$ .

The difference is that in case A the second aspect of the puzzle, which has to do with radiative corrections, is absent. It is clear how 1-loop diagrams are cancelled among themselves due to supersymmetry. On the other hand in case B, the bulk observer would wonder, not only why the central charge is large, but also how the fine cancellations between loop-diagrams and bare values (or counterterms) are achieved.

In other words: if, hypothetically, in our world we found that supersymmetry was restored at a very low scale, say of the order of  $\sim 10^{-3}eV$ , then the cosmological constant problem would become less puzzling. We would perhaps still try to explain the origin of the large hierarchy between the Hubble and the Planck scale, but there would be no puzzle about how the loop diagrams cancel against counterterms: a mechanism, cancellation between bosons and fermions, would guarantee the radiative stability.

### 3. *The Role of Supersymmetry*

From the above it should be clear that we are only interested in non-supersymmetric examples of the AdS/CFT correspondence. In supersymmetric examples we know that bosonic and fermionic loop diagrams cancel each other. In AdS space there may be a small mismatch between the radiative corrections of bosons and fermions<sup>14</sup>, but this at most of the order of the AdS scale  $1/R$ . At higher energy scales, where we can ignore the curvature of AdS space, supersymmetry does indeed lead to cancellations between loop diagrams. Hence loop diagrams may give non-trivial contributions only up to the scale  $M_{SUSY} \sim 1/R$  and we have, in the language of the previous section, that the relevant ratio  $\xi$  is

$$\xi \sim M_{SUSY}^D M_P^{2-D} R^2 \sim (M_P R)^{2-D} \ll 1$$

So there is no need for any other cancellations in perturbation theory, besides those between bosonic and fermionic loops.

Notice however that the cancellation between bosonic and fermionic loop diagrams means that if we consider the double OPE expansion (2.4) for a supersymmetric CFT with a bulk cutoff near the Planck scale we would observe the following fine-tuning: the LHS would be of order  $1/c$ , but on the RHS we would have (besides other terms) partial sums over

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<sup>14</sup> This can also be understood from the CFT point of view: in representations of the superconformal algebra bosonic and fermionic states do not have the same energy, as the supersymmetry operators do not commute with the Hamiltonian on the sphere (i.e. the dilatation operator).

double trace operators of bosons and fermions, where each of these sums is parametrically larger than  $1/c$  but with opposite signs. So even in the supersymmetric case, there are large cancellations in the conformal block expansion but in this case the mechanism responsible for these cancellations is a well understood one, namely supersymmetry.

We do not want to discuss the supersymmetric examples any further. It is well known that supersymmetry might offer an explanation of the c.c. fine-tuning, but this would require a very low scale of supersymmetry breaking, which is experimentally ruled out.

#### *4. The string scale and other cutoffs*

In many examples of AdS/CFT there is a “string scale”  $M_s$ , besides the Planck scale  $M_P$ . Typically in the large  $N$  limit the string scale is of order  $f/R$ , where  $f$  is a factor which depends on other parameters, such as the ’t Hooft coupling. No matter how large  $f$  is, in the conventional large  $N$  limit it does not scale with  $N$ . So in the large  $N$  limit we have  $M_P/M_s \rightarrow \infty$ . Independent of the existence of supersymmetry, the presence of such a string scale, parametrically smaller than the Planck scale, removes the sharp fine-tuning problem. The loop diagrams can be trusted only up to the string scale hence  $M_{cut} \sim \frac{f}{R}$  and we find

$$\xi \sim \frac{f^D}{R^D} M_P^{2-D} R^2 \ll 1$$

which means that while the energy density due to loop diagrams is of order  $M_s^D$ , it is not heavy enough to backreact in the limit  $M_P R \rightarrow \infty$ .

Notice that even in bosonic string theory in flat space, ignoring the tachyon instabilities for a moment, the 1-loop vacuum diagram gives a contribution to the c.c. of order  $M_s^D$  which is parametrically smaller than  $M_P^D$  if  $g_s \ll 1$ .

So the (well-known) conclusion is that a low string scale would resolve<sup>15</sup> the c.c. fine-tuning problem by effectively lowering the cutoff of the effective field theory. But this resolution is not suitable for us, since it would require, in the real world, an unrealistically low string scale (i.e. of the order of  $M_{cut} \sim 10^{-30} M_P \sim 10^{-3} eV$ ).

From the AdS/CFT point of view these observations show that large  $N$  gauge theories in the ’t Hooft limit may be less suitable examples/toy models to study the problem. These theories have duals with a string scale parametrically lower than the Planck scale, as can

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<sup>15</sup> Or more accurately it prevents us from posing a sharp paradox.

be seen from the Hagedorn growth in the spectrum of single trace operators [8], [9], [10]. Hence the 1-loop vacuum energy up to the cutoff of effective field theory is not expected to backreact significantly and we cannot pose a sharp c.c. problem in the bulk.

The same problem exists for bosonic examples of AdS/CFT where the bulk is described by some higher spin theories in AdS, such as [11], [12], [13]. As for the stringy examples, in these models the effective field theory in the bulk has a very low cutoff, of the order of the AdS scale, so we have that the ratio  $\xi \ll 1$  and we cannot pose a sharp c.c. problem.

### 3. CFTs with holographic duals

In order to translate the bulk fine-tuning problem into a statement in the dual CFT we first have to review the basic properties of CFTs with a holographic description. We try to be brief and refer the interested reader to [6] for more details.

The most characteristic properties of such CFTs is that they have large central charge  $c$  and that their spectrum at low conformal dimensions contains a sector with a small number of “generalized free fields”, i.e. operators whose correlators factorize [6]. The factorization has to be understood as a statement which holds in the large  $c$  limit, i.e. up to  $1/c$  corrections. This sector of operators can be naturally represented in terms of a higher dimensional gravitational theory in AdS space. In general this gravitational theory will be very complicated and not well described by Einstein gravity. For example, it may be a highly curved (though weakly coupled) string theory, or higher spin gravity.

One final condition, on top of the ones described in the previous paragraph, guarantees that the bulk theory simplifies significantly and can be described by semi-classical two-derivative gravity: the condition that single-particle operators with spin higher than 2 have conformal dimension parametrically larger than that of the stress energy tensor, as emphasized in [14]. In general let us call  $\Delta_{cut}$  the conformal dimension where the higher spin single-particle operators appear. Semi-classical gravity becomes a good approximation if  $\Delta_{cut} \gg 1$ .

### 3.1. Candidate CFTs with c.c. problem

According to the previous discussions a CFT whose bulk dual displays the c.c. problem must have the following properties:

i) In the first place it must satisfy all the conditions of a holographic CFT i.e. large central charge, few light operators, factorization etc.

ii) The CFT should not be supersymmetric<sup>16</sup>.

iii) The cutoff of the effective theory in the bulk should be high enough so that the 1-loop vacuum energy can backreact significantly i.e.  $M_{cut}^D M_P^{2-D} R^2 \gg 1$ . Translating this into a statement about the conformal dimension  $\Delta_{cut}$  where higher spin/stringy states start to appear we find the condition

$$\Delta_{cut} \gg c^{1/D} \quad (3.1)$$

At the moment we do not know any CFT satisfying all these conditions. Examples of CFTs satisfying conditions i) and ii), but not iii), are non-supersymmetric orbifolds of supersymmetric string backgrounds such as [15], the large  $N$   $O(N)$  models in 3 dimensions or the models discussed in [12], [13]. These theories do not satisfy iii) because the cutoff of effective field theory in the bulk is of the order of the AdS scale, or  $\Delta_{cut} \sim \mathcal{O}(1)$  and not (3.1)<sup>17</sup>.

On the other hand an example of a CFT satisfying i) and iii), but not ii), is the ABJM theory at large  $N$  (and  $k = 1$ ). While the spectrum of operators in this CFT cannot be studied directly from the gauge theory side, we expect from the bulk that the only UV scale is the  $D = 11$  dimensional Planck scale, hence we expect  $\Delta_{cut}$  to satisfy (3.1)<sup>18</sup>.

It would clearly be fascinating to find a CFT where all of these properties i), ii), iii) are simultaneously satisfied. Such a CFT would describe holographically AdS gravity without supersymmetry. Even though such a CFT is not yet known, we will assume that it exists and, guided by the bulk description, try to understand how the c.c. fine-tuning would manifest itself in abstract CFT language.

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<sup>16</sup> Otherwise the cancellation of the zero-point energy is well understood and there is no puzzle to be explained.

<sup>17</sup> Because the higher spin fields appear at conformal dimension of order 1.

<sup>18</sup> Since M-theory is only characterized by the  $D = 11$ -dimensional  $M_P$  (there is no "string scale") we expect that at large  $N$ , the new operators going beyond 11d supergravity will appear at energies of order  $M_P$ . Translating into conformal dimensions it means  $\Delta_{cut} \sim c^{1/(D-2)}$  which is sufficient for (3.1).



### 3.2. Witten diagrams and large $c$ expansion

As we explained in the introduction the bulk c.c. problem can also be phrased in terms of Witten diagrams in the bulk. Hence in order to translate the problem in CFT terms it is crucial to understand the meaning of the Witten diagram expansion from the CFT point of view.

This is a general question, with other possible applications, but which has never been fully clarified, especially to higher orders in  $1/c$ , when loop Witten diagrams become important. In any case, we review what is known so far and we hope the CFT interpretation of the Witten diagram expansion will be understood better in the future.

As argued in [6] and also in [14], [16], [17], the AdS space can be thought of as a convenient way to represent the  $1/c$  expansion around a generalized free CFT. The Witten diagram expansion around a generalized free CFT play a similar role as the Feynman diagram expansion around a standard free field theory.

$$\begin{aligned}
& \text{Witten diagram} = \text{Tree-level exchange} + \sum n \text{ (Dressed exchange)} + \sum n \text{ (Dressed exchange)} \\
& \text{Witten diagram: Circle with } \phi_1, \phi_2, \phi_3, \phi_4 \text{ and internal } \phi_m \\
& \text{Tree-level exchange: } O_1, O_2, O_3, O_4 \text{ with internal } O_m \\
& \text{Dressed exchange 1: } O_1, O_2, O_3, O_4 \text{ with internal } :O_1 \partial^n O_2: \\
& \text{Dressed exchange 2: } O_1, O_2, O_3, O_4 \text{ with internal } :O_3 \partial^n O_4:
\end{aligned}$$

**Figure 6:** Witten diagrams and conformal blocks

In figure 6 we see a basic scalar exchange Witten diagram. At first one might expect that this Witten diagram corresponds in the CFT to the conformal block of the operator  $\mathcal{O}_m$  dual to the exchanged field  $\phi_m$  in the bulk. This is almost, but not quite correct. Besides the conformal block of  $\mathcal{O}_m$  the exchange Witten diagram contains an infinite sum over two-particle<sup>19</sup> operators [18], as shown in the figure. Why are these operators there and what fixes their relative coefficients? The answer is that in theories with a large  $c$  expansion there is a unique way that the basic conformal block  $\mathcal{O}_m$  can be “dressed up” by two-particle conformal blocks into a combination which has a good OPE expansion in the crossed channel [6]. This combination is precisely the exchange Witten diagram depicted above.

This correspondence can be generalized to other kinds of Witten diagrams. More specifically, in order to understand how to deal with loop diagrams we have to address the

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<sup>19</sup> Or “double trace” in the language of large  $N$  gauge theories.

issue of choice of basis of operators. When considering the large  $c$  expansion of CFTs we are not looking at one and fixed CFT but rather at a family of CFTs parametrized by  $c$ . Hence we have a double expansion: we have an expansion in terms of powers of  $1/c$  and an expansion in terms of conformal blocks. In this situation we have to understand how we choose the basis of operators as a function of  $c$ . A natural choice is to work in terms of the “instantaneous” conformal primaries  $\mathcal{O}_i(c)$ . In this basis the 2- and 3-point functions are fixed by conformal invariance and some of the Witten diagrams, such as the self-energy diagrams, are more difficult to interpret.

Another choice of basis, which seems to be closer to what the Witten diagram expansion represents and which was nicely discussed in [17] and also [19], [20], is to try to build the eigenstates (conformal primaries) of the interacting theory by using the Hilbert space of the free one. This is the approach that we usually follow in perturbation theory in quantum mechanics where the interacting eigenstates are written as superpositions of the free ones. From this point of view we start with a free Hamiltonian  $H_0$  describing the propagation of free fields in AdS. This is dual to the dilatation operator in the  $c = \infty$  conformal field theory. We then perform perturbation theory in the small parameter  $g \sim 1/\sqrt{c}$ . The interacting Hamiltonian/dilatation operator has the form

$$H = H_0 + gV_1 + g^2V_2 + \dots \quad (3.2)$$

These operators are constructed to act on the Hilbert space of the generalized free fields i.e. the (low-lying) gauge singlets.

The eigenstates (conformal primaries) of the interacting Hamiltonian  $H$  can be written as superpositions of the eigenstates of the free theory  $H_0$  by doing standard quantum mechanics perturbation theory. This leads to mixing between single- and multi-particle operators. This perturbation theory is related to the Witten diagram expansion, though clearly the details have to be worked out more carefully.

### 3.3. Radiative corrections in AdS/CFT

In most examples of AdS/CFT that we know, the parameter which controls the strength of the bulk interactions is  $1/N$  (or more generally the inverse central charge  $1/c$ ). This parameter is related to the combination  $M_P R$ . This means that once we start considering loop diagrams, we inevitably also have to consider quantum gravity effects. A

related issue is that in many examples the cutoff of the effective (super)-gravity theory is of the order  $M_P$  i.e. it is correlated to the loop-expansion parameter.

Both of these issues can make things a little more complicated. So we will start by first taking the central charge (i.e. the combination  $M_P R$ ), the loop-counting parameter  $g$  and the cutoff  $M_{cut}$  to be (formally) independent quantities. We will consider loop diagrams to determine how they depend on these parameters. In the end, if we want to apply our results to a particular example of AdS/CFT, we have to go back and see how all these quantities are correlated.

In the rest of this short section we will discuss how we can introduce a UV cutoff in an AdS covariant way to regulate the loop momentum integrals and what is its meaning in the dual CFT.

### *Cutoff in AdS*

Let us imagine that we are observers in AdS and that we try to describe physics by some sort of effective field theory. For this we need to introduce a UV cutoff  $M_{cut}$  to regulate the divergences of loop integrals and to parametrize the regime of validity of our effective description.

When doing quantum field theory in curved space it is important to impose the cutoff in a covariant way. In flat space we can think of a cutoff as a maximum energy/momentum allowed to run in the loops. In AdS space there is a global notion of energy, but this cannot be used as a cutoff, due to the large redshift factor between the “center” and the “boundary” of AdS. Instead the cutoff has to be imposed on the energy of virtual particles as measured in a local inertial frame<sup>20</sup>.

In practice if we use a Pauli-Villars regularization method, it will automatically introduce a correct covariant cutoff.

### *Cutoff in the CFT*

At first one might think that the bulk cutoff translates in the CFT into a cutoff in the conformal dimensions of the form  $\Delta_{cut} \sim M_{cut} R$ . However, as we explained above, this is not a covariant cutoff and is in fact inconsistent with conformal invariance. By acting

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<sup>20</sup> We would like to thank S. Minwalla for discussions on these issues.

with the “raising operators”  $P_\mu$  we can indefinitely increase the conformal dimension of an operator. So it is not consistent to impose a sharp conformal dimension cutoff.

Instead what we have to do is to impose a cutoff on the “relative momentum” of many-particle states [17]. This can be done as follows. States with single particles are related to single trace operators  $\mathcal{O}$ . States with two particles are related to “double-trace operators”. When such operators are conformal primaries they have the form

$$: \mathcal{O}(\partial^2)^n \partial_{[1} \dots \partial_{l]} \mathcal{O} :$$

At infinite  $c$  this 2-particle state has conformal dimension  $2\Delta + 2n + l$ . It represents two particles, whose “center of mass” is at rest in the center of AdS and the relative momentum is controlled by  $n, l$ . By considering the descendants of this bound state, we can set the center of mass in motion in AdS.

It is then clear that the cutoff has to be imposed in the CFT by bounding the parameters  $n, l$  in the conformal primaries of two-particle operators (and similarly for three- and higher- particle operators). This type of cutoff is consistent with conformal invariance, or equivalently, covariant with the curved metric of AdS.

## 4. A simple example of a 1-loop diagram

In this section we consider a simple 1-loop diagram in AdS/CFT and try to find its interpretation in the boundary CFT. We are especially interested in the meaning of the UV divergences and of the counterterms introduced to cancel them.

### 4.1. Some background

We consider the AdS/CFT correspondence where we have a  $d$ -dimensional CFT and a  $D = d + 1$  dimensional AdS space. In AdS/CFT single-particle operators  $\mathcal{O}$  of the CFT are associated to fields propagating in the bulk. For scalar operators we have the relation

$$\Delta = d/2 + \sqrt{m^2 + (d/2)^2} \tag{4.1}$$

between the conformal dimension  $\Delta$  of  $\mathcal{O}$  and the mass  $m$  of the dual bulk field  $\phi$ . Correlation functions of  $\mathcal{O}$  can be computed in (bulk) perturbation theory by evaluating Witten diagrams. The bulk-to-boundary propagator for a scalar field of mass  $m$  has the form

$$K_m(x, z) = \frac{\Gamma(\Delta)}{\pi^{d/2} \Gamma(\Delta - d/2)} \left( \frac{z_0}{z_0^2 + (\vec{z} - \vec{x})^2} \right)^\Delta$$

where the vector  $\vec{x}$  denotes a point on the conformal boundary of AdS and  $z = (z_0, \vec{z})$  a point in the bulk in (Euclidean) Poincare coordinates where the metric is

$$ds^2 = \frac{dz_0^2 + d\vec{z}^2}{z_0^2}$$

We also need the bulk-to-bulk propagator

$$G_m(z, w) = \frac{\Gamma(\Delta)}{2^{\Delta+1} \pi^{d/2} \Gamma(\Delta - \frac{d-2}{2})} s^\Delta {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}, \Delta - \frac{d-2}{2}, s^2\right)$$

where

$$s = \frac{2z_0 w_0}{z_0^2 + w_0^2 + (\vec{z} - \vec{w})^2}$$

Internal vertices in the bulk have to be integrated over their position in AdS with the measure

$$\int \frac{d^D z}{z_0^D}$$

#### 4.2. Bulk

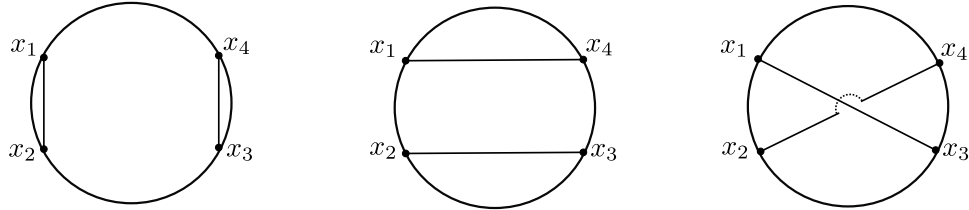
Let us see how we can compute a simple 1-loop diagram in AdS. We consider the  $\phi^4$  theory in the bulk

$$\mathcal{L} = \frac{(\nabla\phi)^2}{2} + \frac{m^2\phi^2}{2} + \frac{g}{4!}\phi^4$$

and for the moment we work in general dimensionality of  $\text{AdS}_D$  where, depending on  $D$ , the interaction can be relevant ( $D < 4$ ), marginally irrelevant ( $D = 4$ ) or irrelevant ( $D > 4$ ). The coupling constant  $g$  has dimensions  $[\text{mass}]^{4-D}$ . We will study the Witten diagrams contributing to the 4-point function of the operator  $\mathcal{O}$  dual to the bulk field  $\phi$  in perturbation theory in  $g$ . We can write

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \mathcal{O}(x_3) \mathcal{O}(x_4) \rangle = \sum_{n=0}^{\infty} W^{(n)}(x_1, x_2, x_3, x_4)$$

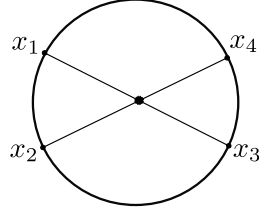
where the contribution  $W^{(n)}$  is of order  $g^n$  and, as usual in quantum field theory, the equality has to be understood as an asymptotic expansion.



**Figure 7:** Disconnected Witten diagrams.

At zeroth order in  $g$  we only have the disconnected components depicted in figure 7. They have a simple factorized form

$$W^{(0)} \sim \frac{1}{|x_1 - x_2|^{2\Delta} |x_3 - x_4|^{2\Delta}} + \text{permutations} \quad (4.2)$$

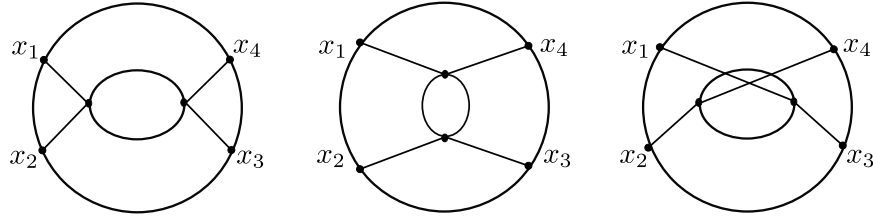


**Figure 8:** Tree level Witten diagram.

At order  $g$  we have to evaluate the tree level diagram of figure 8. This diagram is equal to

$$W^{(1)} \sim g \mathbf{D}_m(x_1, x_2, x_3, x_4) \equiv g \int \frac{d^D z}{z_0^D} K_m(x_1, z) K_m(x_2, z) K_m(x_3, z) K_m(x_4, z)$$

where we have introduced the “**D**-function” for a field of mass  $m$ .



**Figure 9:** 1-loop Witten diagrams.

At order  $g^2$  we have the loop diagrams depicted in figure 9 which are equal to<sup>21</sup>

$$W^{(2)}(x_1, x_2, x_3, x_4) \sim \frac{g^2}{2} \int \frac{d^D z}{z_0^D} \frac{d^D w}{w_0^D} K_m(x_1, z) K_m(x_2, z) G_m(z, w) G_m(z, w) K_m(x_3, w) K_m(x_4, w) \\ + \text{permutations}$$

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<sup>21</sup> We also have loop diagrams with corrections to the propagators of the external legs. Such diagrams are related to corrections of the dimension of the operator  $\mathcal{O}$ . For simplicity we ignore them in this section.

The UV behavior of these integrals can be estimated from the equivalent diagrams in flat space. If we call  $p$  the momentum running in the loop, then the two internal propagators contribute a factor of  $\frac{1}{p^4}$ , at large  $p$ , while the integration over momenta a factor of  $d^D p$ . The degree of divergence of the diagram is  $D - 4$ . Hence we expect this integral to be UV divergent for  $D > 4$ , logarithmically UV divergent for  $D = 4$  and finite for  $D < 4$ . Since these estimates have to do with a UV divergence we expect them to be the same in AdS space.

We can check this in the following way: the divergence is coming from the region  $z \sim w$ . For the moment let us fix  $w$  and expand  $z$  as  $z = w + w_0 \epsilon$ , where  $\epsilon = (\epsilon_0, \vec{\epsilon})$  is a small  $D$ -dimensional coordinate vector in Poincare coordinates. The distance  $s$  can be expanded as

$$s = 1 - \epsilon^2 + \dots$$

where  $\epsilon^2 = \epsilon_0^2 + \vec{\epsilon}^2$  and then the bulk-to-bulk propagator has the following short-distance expansion

$$G_m(w + \epsilon, w) \sim |\epsilon|^{-(D-2)} + \text{subleading}$$

This is indeed the expected leading short-distance behavior of the propagator of a scalar field in flat space.

Considering the integration over  $z$  in a small neighborhood around  $w$  (say down to  $\epsilon \sim \frac{1}{M_{cut}}$ ) we find that the product of two propagators times the measure from  $d^D z$  has a UV divergence of the form  $M_{cut}^{D-4}$ , for  $D > 4$ , and  $\log M_{cut}$  for  $D = 4$ . Hence, in the case  $D < 4$  the integral is finite and can in principle be computed in straightforward way. In  $D \geq 4$  we need to regularize and renormalize.

#### 4.3. Renormalization and counterterms in $D = 4$ .

We now review how this can be done in the case  $D = 4$ , similar analysis can be performed in higher dimensions. We find it practical to adopt a Pauli-Villars regularization method. We introduce a fictitious heavy field  $\Phi$  of mass  $M_{PV}$  and wrong-sign kinetic term. Hence we consider the more general regularized action:

$$\mathcal{L}_{reg} = \frac{(\nabla\phi)^2}{2} + \frac{m^2\phi^2}{2} - \frac{(\nabla\Phi)^2}{2} - \frac{M_{PV}^2\Phi^2}{2} + \frac{g}{4!}(\phi + \Phi)^4$$

The bulk-to-bulk propagator of the field  $\Phi$  is the opposite of that of a field with mass  $M_{PV}$ . In practice, given a loop diagram with external  $\phi$  lines, we simply have to replace the internal bulk-to-bulk propagators by

$$G_m(z, w) \rightarrow G_m(z, w) - G_{M_{PV}}(z, w)$$

In this way we can define the regularized 4-point function as

$$\begin{aligned} W_{1-loop}^{reg}(x_1, x_2, x_3, x_4; M_{PV}) &\sim \frac{g^2}{2} \int \frac{d^4 z}{z_0^4} \frac{d^4 w}{w_0^4} \times \\ &\times K_m(x_1, z) K_m(x_2, z) (G_m(z, w) - G_{M_{PV}}(z, w)) (G_m(z, w) - G_{M_{PV}}(z, w)) K_m(x_3, w) K_m(x_4, w) \\ &+ \text{permutations} \end{aligned}$$

For finite  $M_{PV}$  this integral is convergent. In the limit  $M_{PV} \rightarrow \infty$  it diverges logarithmically, as we expect from flat space. The Pauli-Villars mass  $M_{PV}$  can be qualitatively identified with the UV cutoff  $M_{cut}$ .

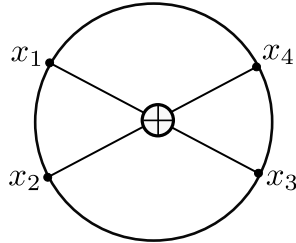
In order to define “renormalized” correlators we have to introduce a counterterm in the Lagrangian, chosen so that the sum of the 1-loop diagram and the counterterm is finite as we send the cutoff to infinity. For the diagram under consideration this can be achieved by adding

$$\mathcal{L}_{c.t.} = \frac{\delta g}{4!} \phi^4$$

where  $\delta g$  has to be chosen to cancel the divergent part of the 1-loop integral. This happens if we take

$$\delta g = -b \frac{g^2}{16\pi} \log \left( \frac{M_{PV}}{\mu} \right)$$

(to fix the overall numerical constant  $b$  we have to keep track of all factors in the intermediate formulas). The scale  $\mu$ , which corresponds to a finite shift of the counterterm, can be fixed by a renormalization condition.



**Figure 10:** 1-loop counterterm.



Putting everything together we find that the 1-loop diagram together with the counterterm give a contribution

$$W^{reg}(x_1, x_2, x_3, x_4; M_{PV}) \sim W_{1-loop}^{reg}(x_1, x_2, x_3, x_4; M_{PV}) - b \frac{g^2}{16\pi} \log\left(\frac{M_{PV}}{\mu}\right) \mathbf{D}_m(x_1, x_2, x_3, x_4)$$

The “renormalized” 4-point function at order  $g^2$  is computed by sending the regulator cutoff to infinity

$$W^{(2)}(x_1, x_2, x_3, x_4) = \lim_{M_{PV} \rightarrow \infty} W^{reg}(x_1, x_2, x_3, x_4; M_{PV})$$

The result is finite and unambiguous up to the choice of the finite term  $\mu$ , which can be fixed by choosing an appropriate renormalization condition/i.e. redefinition of the coupling constant  $g$ .

#### 4.4. Boundary interpretation and fine-tuning

Let us now understand the meaning of these diagrams in the dual CFT. As we mentioned, the interpretation of Witten diagrams in the CFT has never been fully clarified, but it is known that a CFT notion that comes close to a Witten diagram expansion is the expansion of correlators in conformal blocks, see [6] for more details. In a CFT it is natural to analyze a 4-point function by expanding it in a double OPE, say in the  $(12) \rightarrow (34)$  channel, as

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \mathcal{O}(x_3) \mathcal{O}(x_4) \rangle = \sum_{\mathcal{A}} |C_{\mathcal{OO}}^{\mathcal{A}}|^2 \mathbf{G}_{\mathcal{A}}^{12,34}(x_1, x_2, x_3, x_4) \quad (4.3)$$

where the sum runs over all conformal primary operators  $\mathcal{A}$ . The coefficients  $C_{\mathcal{OO}}^{\mathcal{A}}$  depend on the dynamics of the CFT, while  $\mathbf{G}_{\mathcal{A}}^{12,34}(x_1, x_2, x_3, x_4)$  are the conformal blocks i.e. special functions whose form is fixed by kinematics of the conformal group [21]. In theories with weakly coupled holographic duals the Witten diagrams encode certain combinations of conformal blocks with simple behavior under crossing symmetry.

Without going into details, let us illustrate this by first explaining the meaning of the disconnected and tree-level interactions.

When  $g = 0$ , the correlators factorize to products of 2-point functions. If we take the factorized correlator  $W^{(0)}(x_1, x_2, x_3, x_4)$  given in (4.2) and perform an expansion of the

$$\begin{aligned}
& \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + g \text{[Diagram 4]} \\
& + g^2 \left( \text{[Diagram 5]} + \text{[Diagram 6]} + \text{[Diagram 7]} + \text{[Diagram 8]} \right) + \dots \\
& = \sum_{\mathcal{A}} \text{[Diagram 9]}
\end{aligned}$$

**Figure 11:** Expansion of Witten diagrams in conformal blocks.

form (4.3) we find that what runs in-between is the identity operator  $I$ , as well as two-particle conformal primary operators of the form  $:\mathcal{O}(\partial^2)^n \partial_1 \dots \partial_l \mathcal{O}:$ . All these operators come with coefficients of order one, which can be easily computed [21].

The tree level diagram  $W^{(1)}$  can also be expanded in conformal blocks and we find that it corresponds to the exchange of two-particle operators of the form  $:\mathcal{O}(\partial^2)^n \mathcal{O}:$  with coefficients of order  $g$ . As shown in [14] this is the only combination of such (scalar) operators which is consistent with crossing symmetry.

Let us now consider the 1-loop diagram  $W^{(2)}$ , first in the case  $D < 4$  where the diagram is convergent by itself, without any counterterms. In this case it was nicely shown in [22], using the Mellin representation of CFT correlators, that the loop diagram corresponds to the exchange of two-particle operators of the form<sup>22</sup>

$$:\mathcal{O}(\partial^2)^n \partial_1 \dots \partial_l \mathcal{O}: \quad (4.4)$$

These operators come with coefficients of order  $g^2$ . This is intuitively reasonable, cutting the diagram along the loop we see a two-particle state propagating in the middle. The number of derivatives in the conformal primary (4.4) encodes the momentum running in the loop.

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<sup>22</sup> Notice that in this case the exchanged double trace operators are the same for all of these diagrams. If we had a field  $\mathcal{O}'$  running in the intermediate loop then the exchanged double trace operators from the loop diagram would be of the form  $:\mathcal{O}'(\partial^2)^n \partial_1 \dots \partial_l \mathcal{O}'$ .

The integral over the momentum in the bulk translates into an infinite sum over two-particle conformal primaries of this form, with increasing number of derivatives. As we explained, in the case  $D < 4$  this sum is convergent.

In the more interesting case of  $D = 4$  we expect a (logarithmic) UV divergence<sup>23</sup>. We first interpret the diagram with a cutoff  $M_{cut} \approx M_{PV}$ . Expanding it in conformal blocks we find that (up to the cutoff) it corresponds to the exchange of double trace-operators of the same form, with coefficients of order  $g^2$ . However now we notice something very interesting: while each of the contributions of a conformal block is of order  $g^2$  (i.e. small), the sum over conformal blocks of two-particle operators gives a contribution which diverges like  $g^2 \log M_{cut}$ .

In the bulk we corrected this divergence by introducing a “counterterm”, i.e. a diagram with the same structure as the tree level contact interaction, but with a coefficient which is large, of order  $g^2 \log M_{cut}$  (in order to cancel the divergent 1-loop diagram). In the double OPE expansion this means that some of the double trace operators will have to come with very large coefficients in order to cancel the large contribution encountered above from the (partial) sum over conformal partial waves from the loop diagram.

Hence we have arrived at one of the CFT manifestations of the bulk fine-tuning associated to radiative corrections: while the final order  $g^2$  correction to the 4-point function  $W^{(2)}$  is small, the intermediate terms which contribute to the double OPE can be parametrically larger. In particular, partial sums over some of the intermediate operators have very large values and are almost cancelled by large contributions from other operators. All these factors conspire to give a 4-point function which is only of order  $g^2$ .

In this example the expansion in (4.3) would not seem natural<sup>24</sup> to a CFT observer.

## 5. The cosmological constant fine-tuning

After this scalar toy-example let us come back to the fine-tuning of the cosmological constant. As we explained before, the statement that the bulk cosmological constant is small (i.e. that the AdS space is large in units of the Planck scale) means that the correlation functions of gravitons must be appropriately suppressed. In the CFT this

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<sup>23</sup> Of course this sum will be divergent for any  $D > 4$ , we consider  $D = 4$  for simplicity

<sup>24</sup> In the sense that the final value of the sum over  $\mathcal{A}$  is much smaller than individual terms.

means that all connected  $n$ -point functions of stress-energy tensors have a specific scaling. In particular

$$\langle T(x_1) \dots T(x_n) \rangle_{con} \sim c$$

for all  $n$ . Here we have dropped Lorentz indices from the stress-energy tensors and we have suppressed the  $x$ -dependence of the correlator on the RHS. Alternatively we can redefine the stress tensor as  $\tilde{T} = T/\sqrt{c}$  so that the 2-point function is of order 1. Then the statement of a small cosmological constant can be understood as the condition

$$\langle \tilde{T}(x_1) \dots \tilde{T}(x_n) \rangle_{con} \sim c^{(2-n)/2} \quad (5.1)$$

So the question is: if we have a non-supersymmetric CFT with the property (5.1), whose bulk dual has a cutoff at the Planck scale (which in CFT language corresponds to the statement that the generalized free field sector contains few single trace operators up to conformal dimension  $\Delta_{cut} \sim c^{1/(d-1)}$  - see discussion around equation (3.1)), would a CFT observer conclude that the theory seems fine-tuned?

The discussion now continues as before. By considering the 4-point function of gravitons and the contribution to it by, for example, a fermion loop, we find that the 1-loop diagram would lead to a violation of (5.1). This is avoided by a counterterm diagram, of the form of a 4-graviton contact interaction. The two contributions, i.e. the 1-loop diagram and the counterterm, each violate the scaling (5.1), but their sum is parametrically smaller and consistent with (5.1).

The boundary manifestation of the fine-tuning is then parallel to what was discussed in section 2.2 and in section 4 for a scalar field. The correlator  $\langle \tilde{T}(x_1) \tilde{T}(x_2) \tilde{T}(x_3) \tilde{T}(x_4) \rangle$  is of order  $1/c$ . However in the double OPE we would encounter individual conformal blocks, or partial sums over blocks, which are parametrically larger and which in the end cancel among themselves. These conformal blocks correspond to the exchange of double trace operators of the form  $:\mathcal{O}(\partial^2)^n \partial_1 \dots \partial_l \mathcal{O}:$ , where  $\mathcal{O}$  can be the operator dual to any of the propagating fields in the bulk (bosonic or fermionic). Each of the corresponding conformal blocks comes with a coefficient of order  $1/c$  but the sum over  $n, l$  leads to a parametrically larger value. This is cancelled by conformal blocks corresponding to the exchange of operators of the form  $:T \partial \dots \partial T:$ , which are the boundary dual of the c.c. type counterterm Witten diagram depicted in figure 3. Hence in such a CFT the conformal block expansion would seem to be fine-tuned.

## 6. Fine-tuning of scalar masses

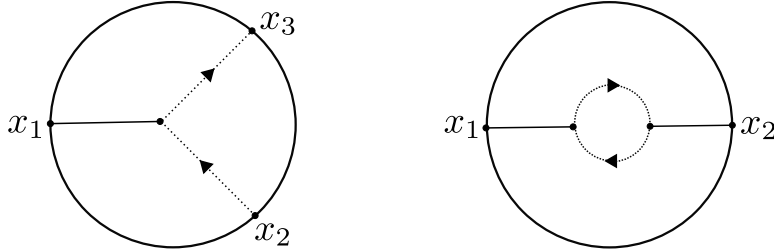
In this section we want to discuss another example of fine-tuning, the one that has to do with the masses of scalar fields. These correspond to relevant operators and in general there are no symmetries to protect them from receiving radiative corrections. This is of course the analogue of the usual hierarchy problem for the Higgs mass in the Standard Model.

### 6.1. The bulk picture

Let us consider  $\text{AdS}_4$  and start with the free theory consisting of a scalar field  $\phi$  of mass  $m$  and a fermion  $\psi$  of mass  $m_f$ . The bulk scalar is dual to a scalar single-particle operator  $\mathcal{O}$  of conformal dimension  $\Delta$  given by (4.1) and the bulk fermion is dual to a fermionic operator  $\Psi$ . We assume that the cutoff of the theory is at a scale  $M_{cut}$ . We turn on a Yukawa-coupling type of interaction

$$V = g \phi \psi \bar{\psi} + h.c.$$

At first order in  $g$  this turns on a 3-point functions between  $\mathcal{O}$  and two fermionic operators  $\Psi$ .



**Figure 12:** Yukawa interaction and scalar self-energy.

At second order in  $g$  we also have a 1-loop correction to the self-energy of the scalar  $\phi$  shown in figure 12. This diagram will lead to a correction to the tree level mass of the boson  $\phi$ . In order to estimate the dependence of the diagram on the UV cutoff  $M_{cut}$  we can use the naive flat-space counting. Each of the fermion propagators contributes a power of  $p^{-1}$ , where  $p$  is the momentum in the loop, and the integration a power of  $p^4$ . So we find that this diagram goes like

$$\delta m^2 \approx g^2 M_{cut}^2 + \text{subleading}$$

This is the usual quadratic divergence of the self-energy of scalars in four dimensions.

This diagram implies that the observed mass of the boson would get a very large contribution of order  $g^2 M_{cut}^2$  from the 1-loop diagram. If however the observed mass is small, then the bulk observer would attribute it to a cancellation between the 1-loop diagram and a “mass counterterm”.

The fine-tuning is the statement that one would have to balance a tree level mass with a 1-loop correction to it, both of which are of order  $g^2 M_{cut}^2$ , in order to produce a very light mass  $m^2$  in the end.

## 6.2. The CFT interpretation

On the CFT side the meaning of the self energy diagram is as follows. In the free theory ( $g = 0$ ) the conformal primaries are  $\mathcal{O}, \Psi$  and their “mutli-particle” composites. These operators have the structure of a freely generated Fock space. When we turn on a small interaction we have to find the new eigenstates of the Hamiltonian (on the sphere) i.e. new eigenvalues (conformal dimensions) and new eigenvectors (conformal primaries) as discussed in section 3.2.

The radiative shift in the mass of the scalar  $\phi$  can be understood as a correction to the energy of the dual state on the sphere, as computed in perturbation theory in  $g$ . In this language the interaction can be written as

$$V = gV_1 + g^2V_2 + \dots \quad (6.1)$$

where  $V_1$  is the interaction from the tree level Yukawa vertex and  $V_2$  contains the counterterms.

The conformal dimension of  $\mathcal{O}$  can be written as

$$\Delta_{\mathcal{O}}(g) = \Delta_{\mathcal{O}}^0 + g\delta\Delta_{\mathcal{O}}^{(1)} + g^2\delta\Delta_{\mathcal{O}}^{(2)} + \dots$$

The correction to the dimension of  $\mathcal{O}$  can be computed to first order in  $g$  as

$$\delta\Delta_{\mathcal{O}}^{(1)} = \langle \mathcal{O} | V_1 | \mathcal{O} \rangle$$

and to order  $g^2$  we have

$$\delta\Delta_{\mathcal{O}}^{(2)} = \sum_{\chi, \Delta_{\chi} \neq \Delta_{\mathcal{O}}} \frac{\langle \mathcal{O} | V_1 | \chi \rangle \langle \chi | V_1 | \mathcal{O} \rangle}{\Delta_{\mathcal{O}} - \Delta_{\chi}} + \langle \mathcal{O} | V_2 | \mathcal{O} \rangle \quad (6.2)$$

The first term comes from the 2nd order perturbation theory expansion from  $V_1$  while the second term from the 1st order perturbation theory from  $V_2$ . The sum over  $\chi$  is over all other states. In our particular case the only states with a nontrivial matrix element are 2-particle states made out of 2 fermions. The sum goes up to the cutoff  $M_{cut}$ .

The statement of the fine-tuning is that the first term in (6.2) is of order  $M_{cut}^2$ , the second term is also of the order  $M_{cut}^2$  but their sum is only of order  $m^2 \ll M_{cut}^2$ .

To summarize: the mass hierarchy problem manifests itself in the CFT as a puzzle of why there are single-particle conformal primary operators with small conformal dimension. The naive perturbation theory of the CFT data predicts a very large correction to the dimensions which can only be cancelled by a fine-tuning of the parameters.

## 7. Naturalness from Holography?

Let us now speculate on the third question that we asked in the introduction, namely how the fine-tuning in AdS might be “resolved” in a natural way in holographic theories. The main observation is that while the description of a large central charge CFT in terms of an effective theory for its light operators (i.e. the generalized free fields - the gauge singlets, in the case of large  $N$  gauge theories) is useful for some purposes, like for making the locality of the bulk dual manifest, it is not how the underlying QFT actually computes the correlation functions.

To give an example, consider a large  $N$  gauge theory in the ’t Hooft limit. Correlation functions of gauge invariant operators can be computed in terms of double-line Feynman diagrams. That such correlation functions are suppressed by the right power of  $1/N$  is obvious in the formalism of the double-line diagrams as ’t Hooft demonstrated. This is the “fundamental” way in which correlators are computed.

We could also try to understand these correlators solely in terms of gauge invariant objects i.e. in terms of exchange of single trace operators. For this we have to take the correlators and expand them in intermediate conformal blocks (for simplicity let us assume that the theory is conformal). Eventually this leads to an effective description of correlators of light operators in terms of gauge singlet collective fields. This description is best suited to be translated to a bulk theory [6]. The point is that, whether the latter description turns out to look fine-tuned or not, we do not really need to worry because we understand what is the underlying mechanism responsible for the smallness of the correlators: it is the

large  $N$  combinatorics of the underlying Feynman diagrams. This mechanism is invisible when we express the correlators in terms of the exchange of gauge singlets.

Similarly, the fine-tuning of light scalar masses appears when the loop corrections to the dimension of single-trace operators is calculated by the (effective) interaction Hamiltonian (6.1). This Hamiltonian acts on the Hilbert space corresponding to the Fock space of the “generalized free fields”. As we mentioned, while it is useful to describe the theory in terms of these variables it is not how the QFT actually determines the dimensions of conformal primaries: the interaction Hamiltonian  $V$  in (6.1) is not the full Hamiltonian of the underlying QFT. The mechanism responsible for the smallness of the conformal dimensions of scalars may be easy to understand in terms of the full Hamiltonian  $H_{QFT}$  but invisible in terms of  $V$ .

### **The case of large $N$ gauge theories**

If large  $N$  gauge theories exhibited fine-tuning in their conformal block expansion, they would be excellent examples where the fine-tuning could be resolved by an underlying mechanism (the large  $N$  counting) which is invisible in the language of the gauge invariant operators. Unfortunately, as mentioned earlier, this is not the case. In large  $N$  gauge theories the conformal block expansion is natural i.e. no fine-tuning is observed in the  $1/N$  expansion in terms of conformal blocks. This is consistent with the fact that large  $N$  gauge theories in the 't Hooft limit are expected to be dual to string theories, i.e. theories where the bulk cutoff is at the order of the “string scale” which is parametrically lower than the planck scale in the large  $N$  limit (this is related to the Hagedorn growth in the spectrum of single-trace operators).

Even though large  $N$  gauge theories do not exhibit the effect that we want to see (i.e. fine-tuning of the conformal block expansion), we can still explore whether they hold any surprises for the IR observer as far as naturalness is concerned. Let us consider  $SU(N)$  Yang-Mills with  $N_f$  fundamental quarks in the 't Hooft large  $N$  limit, keeping  $N_f$  fixed. It is believed that the theory confines and below the strong coupling scale  $\Lambda_{QCD}$  the spectrum of particles consists of gauge singlets i.e. glueballs and mesons. If we were low energy observers we would see all these particles and we might try to construct an effective action describing their interactions. From the low energy point of view we would write the effective action by introducing a field for each particle and the underlying  $SU(N)$  color structure would of course be invisible.



Assuming that the quark masses are small<sup>25</sup>, the masses of the glueballs and mesons will be of order  $\Lambda_{QCD}$ . However we know that couplings between glueballs are suppressed by powers of  $1/N$  while those between mesons by powers of  $1/\sqrt{N}$ . In the large  $N$  limit there is a hierarchy between these couplings, which would probably surprise the low energy observer given that he has no other way of qualitatively distinguishing glueballs from mesons. Moreover, even among meson interactions it turns out that various processes are suppressed by factors of  $1/N$  depending on how many quark lines have to be drawn in the double-line diagrams. This is the so-called “OZI rule”<sup>26</sup>. It implies that in the IR effective action for the mesons certain couplings will be suppressed by additional powers of  $1/N$ . This rule can be easily understood in terms of double-line diagrams (see for example [23]), but would be mysterious for the IR observer who works directly with the mesons.

The reason that this toy model is not fully satisfactory is that the cutoff of the effective field theory is  $\Lambda_{QCD}$  and all interactions between gauge singlets are suppressed by powers of  $1/N$ . Hence the effects of loops (in the IR effective Lagrangian, not the UV nonabelian theory) are quite small and cannot destabilize the tree level values. So while the IR observer would indeed notice large hierarchies between coupling constants and decay rates, he would not have to worry about fine-tuning between loop diagrams and counterterms.

## 8. Discussions

We discussed the conditions under which a bosonic CFT with large central charge has a holographic dual with a sharp “cosmological constant problem”. We argued that in such a CFT the bulk fine-tuning manifests itself in terms of an apparent fine-tuning of the  $1/N$  expansion of correlators in conformal blocks. Finally we proposed the idea that while this fine-tuning may be visible when the correlators are expressed in terms of the exchange of conformal primaries, it may disappear if the correlators are written in terms of the fundamental fields of the underlying QFT.

If this last possibility is true and if we extrapolate it to our world, it would imply that the mechanism responsible for the fine-tuning may be invisible in terms of the low energy

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<sup>25</sup> By this we mean of the order  $\Lambda_{QCD}$ . If the quark masses are significantly smaller then we will have approximate  $SU(N_f) \times SU(N_f)$  symmetry, whose axial part is spontaneously broken by chiral symmetry breaking. The resulting (almost) massless mesons will dominate the IR effective Lagrangian.

<sup>26</sup> Okubo-Zweig-Iizuka rule.

fields (the graviton and the fields of the Standard Model) but may be simple to understand in terms of the fundamental fields (i.e. in the “holographic dual of the universe”).

How can we check if this possibility has any chance of being true, at least in the context of AdS/CFT? The most optimistic scenario would be to find a specific CFT whose AdS dual has a sharp c.c. problem and to check how exactly the fine-tuning is resolved on the boundary. Of course it would be very nice to have such an example for many other reasons, as it would be a non-perturbatively well defined theory of AdS quantum-gravity without supersymmetry<sup>27</sup>.

A somewhat easier goal is to find a toy-model which would illustrate the general logic in a simpler setting. For instance, it would be nice to find an example of a QFT where a prediction of effective field theory (related to the smallness of relevant operators) is invalidated due to some microscopic mechanism which is invisible in terms of the effective degrees of freedom in the IR. Or, more likely, a quantum system (not necessarily a full-fledged QFT) where certain computations in terms of effective/collective degrees of freedom of the system give misleading answers relative to the exact computation in terms of the fundamental degrees of freedom. We hope to report on this in the future.

The idea that the cosmological constant fine-tuning may be an artifact of using emergent rather than fundamental variables has also been discussed by G.E. Volovik [24] in a rather different context. We would like to thank E. Verlinde for bringing these papers to our attention and for many other discussions regarding his recent work [25].

Let us finish with some general observations on a related topic, which was also emphasized in [17].

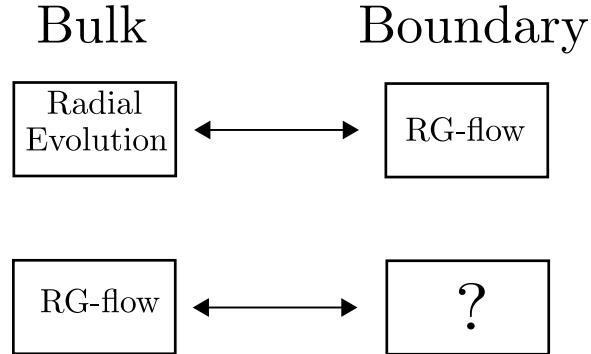
### *The meaning of effective field theory and RG-flow in the bulk*

After all, in order to find a satisfactory explanation of the c.c. fine-tuning we have to understand why the predictions of effective field theory are not completely reliable in some situations. The idea of naturalness is based on the paradigm of Wilsonian RG-flow, i.e. that the low-energy effective theory can be derived by doing RG-flow from a UV theory. From this point of view it is indeed difficult to explain small coefficients of relevant operators. However the Wilsonian intuition seems to be in some tension with

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<sup>27</sup> As we explained before, in all known bosonic examples of AdS/CFT the bulk theory has a very low cutoff and thus does not have a sharp c.c. problem.

the idea of holography where the bulk is an emergent structure: the low-energy effective field theory in the bulk is not necessarily determined by RG-flow from a UV theory in the bulk, but holographically from the boundary QFT, though there may also be some partial consistency condition with the bulk RG-flow. In order to understand this better it would be interesting to develop further the boundary interpretation of the bulk RG-flow.



**Figure 13:** It is known that the usual RG-flow in the boundary QFT can be related to the radial evolution in the bulk. What is the boundary meaning of the bulk RG-flow?

In the AdS/CFT literature there have been extensive discussions about the relation of the RG-flow in the boundary field theory to the radial evolution in the bulk. This is not what we are talking about. Here we are referring to RG-flow in the bulk and its meaning in the dual CFT. Even if the boundary theory is an exact conformal field theory (no mass scale), the theory in the bulk obviously has certain length scales, such as the Planck mass and the masses of other particles in AdS. The masses of these fields are dual to the conformal dimensions of single-particle operators in the dual CFT. In the bulk it is perfectly normal to consider the RG-flow in the effective field theory. This translates into some kind of flow in the direction of conformal dimensions (see also [17]). This flow is distinct from the standard RG-flow in the field theory, which is trivial for CFTs. Presumably this new type of flow only makes sense in CFTs with a holographic dual, or in other words, in the sector of the CFT described by the generalized free fields. It might be interesting to develop this further, perhaps assuming (for simplicity) that there is a regime where non-gravitational interactions in the bulk become parametrically more important than gravity.

## Acknowledgments

We would like to J. de Boer, S. El-Showk, R. Emparan, B. Fiol, N. Iizuka, G. Pastras, J. Penedones, S. Raju, R. Rattazzi, S. Rychkov, M. Shigemori and especially S. Minwalla and E. Verlinde for discussions. We would like to thank the Perimeter Institute for Theoretical Physics for hospitality during the “Back to the Bootstrap” Workshop. We also thank the theory group in Santiago de Compostela and the group in Barcelona for hospitality during the completion of this work.

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